

# STRUCTURE FORMATION IN THE UNIVERSE FROM TEXTURE INDUCED FLUCTUATIONS

Ruth Durrer and Zhi-Hong Zhou

Universität Zürich, Institut für Theoretische Physik,  
Winterthurerstrasse 190,  
CH-8057 Zürich, Switzerland

## Abstract

The topic of this letter is structure formation with topological defects. We first present a partially new, fully local and gauge invariant system of perturbation equations to treat microwave background and dark matter fluctuations induced by topological defects (or any other type of seeds). We show that this treatment is extremely well suited for linear numerical analysis of structure formation by applying it to the texture scenario. Our numerical results cover a larger dynamical range than previous investigations and are complementary since we use substantially different methods.

PACS numbers: 98.80-k 98.80.Hw 98.80C

Despite great effort and considerable progress, the problem of structure formation in the universe remains basically unsolved. Observations show that the fluctuation spectrum on the large scales observed by COBE should be not very far from scale invariant [1, 2, 3]. This has been considered as great success for inflationary models which predict a scale invariant fluctuation spectrum. In this letter we consider an alternative class of models which also yield a scale invariant spectrum of Cosmic Microwave Background (CMB) fluctuations: Models where perturbations are seeded by global topological defects which can form during symmetry breaking phase transitions in the early universe [4]. To be specific, we consider texture,  $\pi_3$ -defects which lead to event singularities in four dimensional spacetime [5]. A common feature of global topological defects is the behaviour of the energy density in the scalar field which scales like  $\rho_T \propto 1/(at)^2$  and thus represents always the same fraction of the total energy density of the universe.

$$\rho_T/\rho \sim 8\pi G\eta^2 \equiv 2\epsilon , \quad (1)$$

where  $\eta$  determines the symmetry breaking scale. The background spacetime is a Friedmann–Lemaître universe with  $\Omega = 1$ . We choose conformal coordinates such that

$$ds^2 = a^2(-dt^2 + \delta_{ij}dx^i dx^j) .$$

Numerical analyses of CMB fluctuations from topological defects on large scales have been performed in [6, 7]. A spherically symmetric approximation is discussed in [8]. Results for intermediate scales are presented in [9]. All these investigations (except the quite rough spherically symmetric calculation [8]) use linear cosmological perturbation theory in synchronous gauge and take into account only scalar perturbations. In this letter we derive a fully gauge invariant and local system of perturbation equations. The (non local) split into scalar, vector and tensor modes on the hypersurfaces of constant time is not performed. We solve the equations numerically in a cold dark matter (CDM) universe with global texture. In this letter, we present the main results. In a longer paper we give detailed derivations of the equations and fully describe our numerical methods [10]. Since there are no spurious gauge modes in our initial conditions, there is no danger that these may grow in time and much of the difficulties to choose correct initial conditions (see e.g. [7]) are removed.

We find that previously ignored vector and tensor fluctuations contribute approximately 25 % to the total microwave background anisotropies (see Fig. 1).

We calculate the microwave background anisotropies on angular scales which are larger than the angle which subtends the horizon scale at decoupling of matter and radiation,  $\theta > \theta_d$ . For  $\Omega = 1$  and  $z_d \approx 1000$

$$\theta_d = 1/\sqrt{z_d + 1} \approx 0.03 \approx 2^\circ . \quad (2)$$

It is therefore sufficient to study the generation and evolution of microwave background fluctuations after recombination. During this period, the photons are decoupled from baryonic matter and free stream, influenced solely by cosmic gravitational

redshift and by perturbations in the gravitational field (if the medium is not reionized). The photon distribution function which lives in seven dimensional relativistic phase space  $P_0\mathcal{M} = \{(x, p) \in T\mathcal{M} | g(x)(p, p) = 0\}$ , obeys Liouville's equation

$$X_g(f) = 0 . \quad (3)$$

In a tetrad basis  $e_\mu$ ,  $X_g$  is given by (see e.g. [11])

$$X_g = (p^\mu e_\mu + \omega_\mu^i(p) p^\mu \frac{\partial}{\partial p^i}) , \quad (4)$$

where  $\omega_\mu^\nu$  are the connection 1-forms of  $(\mathcal{M}, g)$  in the basis  $e^\mu$ .

The metric of a perturbed Friedmann universe with  $\Omega = 1$  is given by  $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$  with

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu}) = a^2\tilde{g}_{\mu\nu} , \quad (5)$$

where  $(\eta_{\mu\nu}) = \text{diag}(-, +, +, +)$  is the flat Minkowski metric and  $|h_{\mu\nu}| \ll 1$  is a small perturbation. We now use the fact that the motion of photons is conformally invariant. Taking into account the different affine parameters, (3) is equivalent to

$$(X_{\tilde{g}}f)(x, ap) = 0 . \quad (6)$$

If  $\bar{e}^\mu$  is a tetrad in Minkowski space,  $e_\mu = \bar{e}_\mu + (1/2)h_\mu^\nu \bar{e}_\nu$  is a tetrad w.r.t the perturbed geometry  $\tilde{g}$ . For  $(x, \bar{e}_\mu p^\mu) \in \bar{P}_0$ , thus,  $(x, e_\mu p^\mu) \in P_0$ . We can therefore define the perturbation of the distribution function  $F$  by

$$f(x, p^\mu e_\mu) = \bar{f}(x, p^\mu \bar{e}_\mu) + F(x, p^\mu \bar{e}_\mu) . \quad (7)$$

Furthermore, we set  $p^i = p\gamma^i$ , with  $p^2 = \sum_{i=1}^3 (p^i)^2$  and  $v = ap$ . Liouville's equation for  $f$  then yields a perturbation equation for  $F$ . We choose the natural tetrad  $e_\mu = \partial_\mu + (1/2)h_\mu^\nu \partial_\nu$ . Using (3,4) and (6) we obtain

$$(\partial_t + \gamma^i \partial_i)F = -[\dot{H}_L + (A_{,i} + \frac{1}{2}\dot{B}_i)\gamma^i + (\dot{H}_{ij} - B_{i,j})\gamma^i \gamma^j]v \frac{df}{dv} , \quad (8)$$

where we have parametrized

$$(h_{\mu\nu}) = \begin{pmatrix} 2A & 2B_i \\ 2B_i & 2H_L \delta_{ij} + 2H_{ij} \end{pmatrix} , \quad (9)$$

with  $H_i^i = 0$ .

We now define

$$m = (1/4) \frac{4\pi}{\rho_r a^4} \int F v^3 dv .$$

$4m$  is the fractional perturbation of the brightness  $\iota$ ,

$$\iota = a^{-4} \int f v^3 dv .$$

Setting  $\iota = \bar{\iota}(T(\gamma, x))$ , one finds that  $m$  corresponds to the fractional perturbation in the temperature,

$$T(\gamma, x) = \bar{T}(1 + m(\gamma, x)) . \quad (10)$$

A more explicit derivation of (10) is given in [12]). Integrating (8)  $v^3 dv$ , we obtain

$$\partial_t m + \gamma^i \partial_i m = \dot{H}_L + (A_{,i} + \frac{1}{2} \dot{B}_i) \gamma^i + (\dot{H}_{ij} - B_{i,j}) \gamma^i \gamma^j . \quad (11)$$

It is well known that the equation of motion for photons only couples to the Weyl part of the curvature (null geodesics are conformally invariant). The r.h.s. of (11) is given by first derivatives of the metric only which could at most represent integrals of the Weyl tensor. To obtain a local, non integral equation, we thus rewrite (11) in terms of  $\Delta m$ . In fact, defining

$$\chi = \Delta m - (\Delta H_L - \frac{1}{2} H_{,ij}^{ij}) - \frac{1}{2} \Delta B_i \gamma^i ,$$

(11) yields for  $\chi$  the equation of motion

$$(\partial_t + \gamma^i \partial_i) \chi = -3 \gamma^i \partial^j E_{ij} - \gamma^k \gamma^j \epsilon_{klj} \partial_l B_{ij} , \quad (12)$$

where  $\epsilon_{kli}$  is the totally antisymmetric tensor in three dimensions,  $E_{ij}$  and  $B_{ij}$  are the electric and magnetic part of the Weyl tensor. The spatial indices in this equation are raised and lowered with  $\delta_{ij}$  and therefore no care is taken in the index position. Double indices are summed over irrespective of their position.

In eqn. (12) the contribution from the electric part of the Weyl tensor does not contain tensor perturbations. On the other hand, scalar perturbations do not induce a magnetic gravitational field. The second contribution to the source term in (12) represents a combination of vector and tensor perturbations. If vector perturbations are negligible, the two terms on the r.h.s of (12) therefore represent a split into scalar and tensor perturbations which is local.

In terms of metric perturbations, the electric and magnetic part of the Weyl tensor are given by (see, e.g. [13])

$$E_{ij} = \frac{1}{2} [\Delta_{ij} (A - H_L) - \dot{\sigma}_{ij} - (\Delta H_{ij} + \frac{2}{3} H_{lm}^{lm} \delta_{ij}) - H_{il,j}^l - H_{jl,i}^l] \quad (13)$$

$$B_{ij} = \frac{1}{2} (\epsilon_{ilm} \sigma_{jm,l} + \epsilon_{jlm} \sigma_{im,l}) , \quad (14)$$

$$\text{with } \sigma_{ij} = \frac{1}{2} (B_{i,j} + B_{j,i}) - \frac{1}{3} \delta_{ij} B_l^l - \dot{H}_{ij} \quad \text{and} \quad \Delta_{ij} = \partial_i \partial_j - (1/3) \delta_{ij} \Delta .$$

Since the Weyl tensor of Friedmann Lemaître universes vanishes, the rhs of (12) is manifestly gauge invariant. (This is the so called Stewart lemma [16].) Therefore also the variable  $\chi$  is gauge invariant.

The general solution to (12) is given by

$$\chi(t, \mathbf{x}, \gamma) = \int_{t_i}^t S_T(t', \mathbf{x} + (t' - t)\gamma, \gamma) dt' + \chi(t_i, \mathbf{x} + (t_i - t)\gamma, \gamma) , \quad (15)$$

where  $S_T$  is the source term given on the rhs of (12).

The electric and magnetic part of the Weyl tensor are determined by the perturbations in the energy momentum tensor via Einstein's equations. We assume that the source for the geometric perturbations is given by the scalar field and dark matter. The contributions from radiation may be neglected. Furthermore, vector perturbations of the dark matter (which decay quickly) are neglected. The divergence of  $E_{ij}$  is then determined by (see, [13] or [14])

$$\partial^j E_{ij} = -8\pi G \rho_{DM} \gamma^i D_i - 8\pi G (\partial_i \delta T_{00} + 3(\frac{\dot{a}}{a}) \delta T_{0i}) + 12\pi G \partial^j \tau_{ij} , \quad (16)$$

where

$$\tau_{ij} \equiv T_{ij} - (a^2/3) \delta_{ij} T_l^l = \tau_{ij}^{(texture)} = \phi_{,i} \phi_{,j} - (1/3) \delta_{ij} (\nabla \phi)^2 ,$$

$$\delta T_{0j} = \delta T_{0j}^{(texture)} = \dot{\phi} \phi_{,j} ,$$

$$\delta T_{00} = \delta T_{00}^{(texture)} = \frac{1}{2} ((\dot{\phi})^2 + (\nabla \phi)^2) ,$$

and  $D_j$  is a gauge invariant perturbation variable for the density gradient (see [13, 14, 10]). For scalar perturbations  $D_j = \partial_j D$ . The evolution equation for the dark matter density perturbation is given by

$$\ddot{D} + (\frac{\dot{a}}{a}) \dot{D} - 4\pi G a^2 \rho_{DM} D = 8\pi G \dot{\phi}^2 . \quad (17)$$

The equation for  $B_{ij}$  is more involved. A somewhat cumbersome derivation [10] yields

$$\ddot{B}_{ij} + 3(\frac{\dot{a}}{a}) \dot{B}_{ij} - \triangle B_{ij} = 8\pi G \mathcal{S}_{ij}^{(B)} , \quad (18)$$

$$\text{with } \mathcal{S}_{ij}^{(B)} = \epsilon_{lm(i} \delta T_{0l,j)m} - (\dot{a}/a) \epsilon_{lm(i} \tau_{j)l,m} .$$

Here  $(i...j)$  denotes symmetrization in the indices  $i$  and  $j$ .

To these equations we have to add the evolution equation of the scalar field,

$$\ddot{\phi} + 2(\dot{a}/a) \dot{\phi} - \triangle \phi = a^2 \lambda \phi (\phi^2 - \eta^2) . \quad (19)$$

We have solved the closed hyperbolic system (12, 16, 17, 18, and 19) numerically on a  $192^3$  grid for different initial conditions on a NEC-SX3 computer at the Centro Svizzero di Calcolo Scientifico (CSCS). The numerical methods employed and the

different tests of our programs are described in [10]. Here we just want to present the main results.

Since on subhorizon scales gauge dependent and gauge invariant variables do not differ substantially we can interpret the variables  $D$  and  $\chi$  by

$$D = (\delta\rho/\bar{\rho}) = \delta \quad \text{and} \quad \chi = \Delta(\delta T/\bar{T}) .$$

Using fast Fourier transforms we calculate the spectrum  $P(k) = |\delta(k)|^2$  and  $\delta T/\bar{T}$ , which we then expand in spherical harmonics

$$(\delta T/\bar{T})(t_0, \mathbf{x}, \boldsymbol{\gamma}) = \sum_{lm} a_{lm}(\mathbf{x}) Y_{lm}(\boldsymbol{\gamma}) . \quad (20)$$

As usual we assume that the average over different observer positions coincides with the ensemble average and determine

$$c_l = \frac{1}{(2l+1)N_x} \sum_{m,x} |a_{lm}(\mathbf{x})|^2 . \quad (21)$$

We have performed 10 simulations on a  $192^3$  grid with about 100 different observer positions for each simulation. The average harmonic amplitudes with  $1\sigma$  variance are shown in Fig. 2. The low order multipoles depend strongly on the random initial conditions (cosmic variance), like in the spherically symmetric simulation [8]. From Fig. 2 it is clear that the texture scenario is compatible with a scale invariant spectrum. The main difference of the currently favored scenarios with inflation induced perturbations lies in the distribution of fluctuations which is non Gaussian in models with topological defects. The pixel distribution for  $\Delta T/T$  in the sky is negatively skewed. Our quadrupole amplitude is given by

$$Q = (0.53 \pm 0.16)\epsilon$$

To reproduce the COBE amplitude  $Q_{COBE} = (0.6 \pm 0.1)10^{-5}$  [3], we have to normalize our spectrum by choosing the phase transition scale  $\eta$

$$\epsilon = 4\pi G\eta^2 = (1.1 \pm 0.5)10^{-5} . \quad (22)$$

This value is comparable with the value of  $\epsilon$  obtained in [6, 7].

The power spectrum of dark matter density fluctuations is shown in Fig. 3. To be compatible with observations ( $\sigma_8 \approx 1$ ), we have to introduce a somewhat high bias factor of

$$b \approx 4 \pm 2 .$$

(The bias factor takes into account that the observed clustering of light does not necessarily coincide with the clustering of the underlining dark matter distribution.) Observations and simulations of nonlinear clustering of dark matter and baryons [17] hint that a bias factor  $b=2 - 2.5$  might be reasonable.

In this letter we have presented a closed system of cosmological perturbation equations which are not plagued by gauge modes and which is well suited for numerical analysis. Our numerical results are consistent with previous investigations

[6, 7, 8], indicating that the texture scenario of structure formation yields somewhat too much power on small scales.

**Acknowledgement** We thank the staff at CSCS, for valuable support. Especially we want to mention Andrea Bernasconi, Djordic Maric and Urs Meier.

## References

- [1] G.F. Smoot et al., *Ap. J.* **396**, L1 (1992).
- [2] R. Scaramella and N. Vittorio, *Mon. Not. R. Astron. Soc.* **263**, L17 (1993).
- [3] C.L. Bennett et al., *COBE Preprint 94-01*, submitted for publication in *Ap. J.* (1994).
- [4] T.W.B. Kibble *J. Phys.* **A9**, 1387 (1976).
- [5] N. Turok *Phys. Rev. Lett.* **63**, 2625 (1989).
- [6] D. Bennett and S.H. Rhie, *Astrophys. J.* **406**, L7 (1993).
- [7] U.-L. Pen, D.N. Spergel and N. Turok *Phys. Rev.* **D49**, 692 (1994).
- [8] R. Durrer, A. Howard and Z.H. Zhou *Phys. Rev.* **D 49**, 681 (1994).
- [9] D. Coulson, P. Ferreira, P. Graham and N. Turok, *Nature* in press (1994).
- [10] R. Durrer and Zhi-Hong Zhou, in preparation.
- [11] J.M. Stewart, *Non-Equilibrium Relativistic Kinetic Theory*, Springer Lecture Notes in Physics, Vol. 10, ed. J. Ehlers, K. Hepp and H.A. Wiedenmüller (1971).
- [12] R. Durrer, *Fundamentals in Cosmic Physics*, in press (1994).
- [13] J.C.R Magueijo, *Phys Rev.* **D46** 3360 (1993).
- [14] M. Bruni, P.K.S. Dunsby and G. Ellis, *Ap. J* **395**, 34 (1992).
- [15] H. Kodama and M. Sasaki *Prog. Theor. Phys. Suppl* **78**, 1 (1984).
- [16] J.M. Steward and M. Walker *Proc. R. Soc. London* **A341**, 49 (1974).
- [17] R. Cen and J. Ostriker, *Ap. J. Lett.* **399**, L113 (1992).

## Figure Captions

**Fig. 1**

The amplitude of the electric and magnetic source terms to the photon equation if motion are shown as a function of wavenumber  $k$  in arbitrary scale. On very large scales the magnetic part contributes about 1/4 decaying to roughly 1/10 on small scales ( $\bar{E} = \frac{1}{3} \sum_i (\partial^j E_{ij})^2$ ,  $\bar{B} = \frac{1}{6} \sum_{ij} (\epsilon_{jlk} \partial^l B_{ki})^2$ ).

**Fig. 2**

The harmonic amplitudes  $l(l+1)c_l$  are shown with  $1\sigma$  error bars. The slight rise at large  $l$  is due to the finite size of the texture core. No smoothing is applied.

**Fig. 3**

The power spectrum of the CDM fluctuations induced by texture. (The vertical scale is arbitrary.)



This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/astro-ph/9407027v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/astro-ph/9407027v1>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/astro-ph/9407027v1>